

EML Webinar overview: New developments in shell stability

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ABSTRACT

The stability of structures continues to be scientifically fascinating and technically important. Shell buckling emerged as one of the most challenging nonlinear problems in mechanics in the middle of the last century when it was first intensively studied. The subject has returned to life motivated not only by structural applications but also by developments in the life sciences and in the field of soft materials. The challenge is that shell structures are susceptible to dramatic load-carrying reductions due to relatively small imperfections in their geometry. Imperfections must be factored into buckling load estimations. Recent work on spherical shells subject to external pressure will be used to illustrate some of the new developments in shell stability. Buckling mode localization, imperfection-sensitivity, energy barriers for stability, and probing to establish the stability landscape will be discussed. EML Webinar speakers and videos are updated at <https://imechanica.org/node/24098>.

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Extended Summary with references: Thin-walled shell structures are extremely effective at carrying large compressive stresses, but they tend to be susceptible to a catastrophic loss in load carrying capacity when they buckle and they are highly imperfection-sensitive [1]. The two best known examples are cylindrical shells under axial compression and spherical shells under external pressure. Experimental data collected in the 1960's for the cylindrical shell under axial compression is shown in Fig. 1. The empirical knockdown factor in Fig. 1 for this shell/loading combination proposed by NASA has been included in most design codes [2] and is still used. Efforts are underway in the aerospace communities in China, Europe and the US to update existing design codes for shell buckling to take account of computational, experimental and manufacturing advances occurring in recent decades which promise to enable less conservatively designed structures [3–11]. These efforts propel some of the resurgence of interest in shell buckling. Additional motivation has emerged in the life sciences in the study of microscopic shell structures of entities like pollen grains, viruses and colloidosomes and in the burgeoning field of soft materials where it has been possible to conduct benchtop buckling experiments that would be inconceivable for metallic shells [12–17].

There is a rich array of nonlinear phenomena hidden beneath the data in Fig. 1, some of which has only been exposed in recent years. For spherical shells the stage was set by the early work of von Karman and Tsien [18] and application of Koiter's [19–22] general theory of elastic stability which directly connected imperfection-sensitivity to the post-buckling behavior of the perfect shell. The analytical methods of the period prior

to 1970 did not permit consideration of realistic imperfection shapes nor realistic imperfections amplitudes, and the numerical methods of that era were not yet sufficiently developed to fill the gap. A noticeable lull in research in shell buckling for more than three decades following the initial progress during which powerful numerical codes were developed capable of simulating shell buckling with realistic imperfections and, at the same time, new understanding of nonlinear phenomena and methods to deal with them emerged in the nonlinear dynamics community. These technological advances coupled with the motivations noted earlier opened the research opportunities and progress which will be cited next.

The early methods generally focused on deformation modes and geometric imperfections in the shape of the classical buckling modes of the perfect shell, and these modes tend to extend over the entire shell. While there was a realization that localized modes and imperfections might be important, the methods available were not effective in coping with them. The importance of localization in shell buckling has been demonstrated clearly in more recent work, both as a post-buckling phenomenon for perfect shells and in the representation of more realistic geometric imperfections [23–33]. Experiments with elastomeric spherical shells manufactured to have precisely formed and characterized local dimple-shaped geometric imperfections have provided quantified agreement between experiment and accurate imperfection-sensitivity simulations [16]. Furthermore, this confirmation for localized imperfections is convincing evidence that the world will almost certainly have to live with the current low knockdown factor of about 0.2 for unstiffened thin spherical shells under external pressure unless it is demonstrated that middle surface departures from the perfect geometry can be held to very demanding tolerances.

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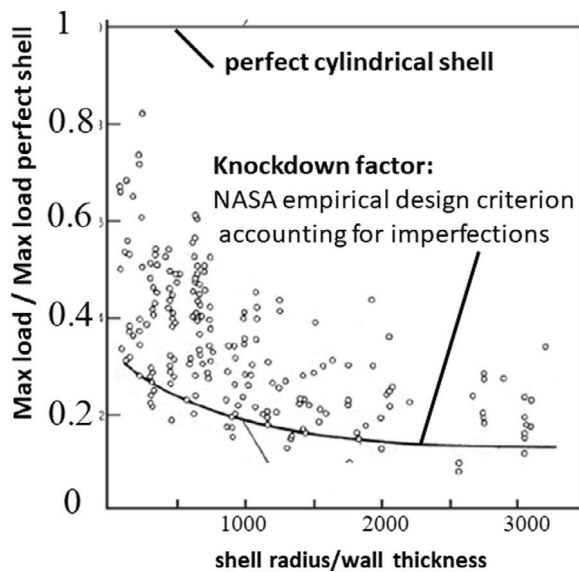


Fig. 1. Experimental data from many investigators for the elastic buckling of cylindrical shells under axial compression collected by NASA in the 1960's. Large scatter is evident with most shells buckling at loads well below the prediction for the perfect shell. To deal with this, NASA [2] in 1965 proposed that the design load for buckling be assigned using the knockdown factor shown in the figure multiplying the prediction for the perfect shell, an approach still used today.

An important concept in the thermodynamics of materials is the energy barrier between different states of a material used to access the role of thermal actuation in causing phase changes. Recently, the energy barrier between the unbuckled state of a loaded structure and buckled states at the same load has been put forward as a possible measure of the robustness, or lack thereof, of the structure to buckling [32,34–39]. For all but extremely small structures, the disturbances pushing a structure over the barrier would not be thermally actuated motion but most likely impacts or local forces of various types. Another recent development is the use of probing a loaded shell to explore, and measure, the stability landscape of the structure [37,38,40–43] and thereby determine the proximity of the shell to buckling. Potentially, probing could provide a non-destructive method for measuring buckling loads. Promising experimental investigations along these lines have been conducted on cylindrical Coke cans [37] and on spherical shells [38]. One can also use probing, either experimentally or as a computational tool, to gain insights into the nonlinear behavior of a loaded structure that is not revealed by standard buckling approaches, as has been convincingly demonstrated for a lightweight planar space structure constructed from thin coilable shell members [43].

We close by noting that attention in this survey of recent developments has been limited to unstiffened monocoque shells with no consideration of the role of stiffening or of how the new developments will impact composite shells and sandwich shells. Many, if not most, light weight aerospace shells are now stiffened or composite shell structures. The application of the new developments surveyed here to these shells is in its infancy and is likely to highly productive.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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